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Markov Chain Modelling of Safety Incident Data: A Veritable Decision Support Tool

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ABSTRACT

The rate of industrial accident occurrence has remained a perennial challenge. This paper points to the need for the application of Markov chain model in unwrapping the deeper meanings buried in the safety incident data that mere descriptive statistics can hardly furnish. In line with the study design, a 16-year data was obtained from Nigeria Gas Company, a subsidiary of Nigeria National Petroleum Corporation (NNPC). The historical incidence records were characterized and proved to possess a note of stochastic regularity that fits into a Markov Chain Model. A twenty state transition was used for the study, namely: fatality, third party fatality, permanent disability for example. Result emanating from the study reveals that subjects make about thirteen habituations among various states in the organization before being absorbed in any ten absorbing states with a standard deviation of 12. Remarkably, 70.5% of the field workers in the organization had noteworthy severe medical treatment case. In conclusion, the Markov Chain Model was able to identify states such as unsafe acts and unsafe conditions transitions to have influenced incident levels the most in the organization. This study has also shown that Markov Chain model can be successfully applied to industrial accident data, unveiled significant visits, habituations which the organization can explore in optimizing their injury prevention programme and ensures field staff safety.

1. Introduction

In spite of the multi-strategic approach to nipping industrial accident occurrences to the base in most factories, the phenomenon appears progressively unabating of which the oil and gas industry situation is no exception. This is attributed to manager's lack of commitment and dependence on mere descriptive statistics as a major tool for analyzing and monitoring accident statistical data to establish trend and patterns. Moreover, [1] corroborated this fact, according to

them only with descriptive statistics it is not possible to carry out an in depth review of the causes of accidents. Therefore, statistical methodologies have been improving which have enabled better safety design and policy improvement. Hence, the need for new ways and better robust, rigorous engineering models such as the Markov chain model which is considered effective for analyzing accident statistical historic records in order to unveil deeper meanings embedded in such accident data aimed at reducing accident risk occurrence that descriptive statistics can hardly provide. The Markov chain model has the capability of not just establishing trend and patterns but unwrapping insightful transitions, habituation workers in an organization undergo, movements, visit of certain state events before they are absorbed by a major life threatening state.

However, accident is yet under reported, arbitrary, and in some cases in Nigeria highly customized. According to [2] fatality, injury and accident rates are increasing and management commitments to accident remain poor and this arguably contributes largely to accident causation accounting for 91.3% contributory factor. It has been observed, over the years, that manager's of oil and gas industries are more reactive to health and safety issue as they occur than before they are likely to happen. The foregoing became obvious when investigations into several episodic accidents such as the Piper Alpha disaster (1988), Bhopal Gas plant disaster (1984), Chernobyl Nuclear power plant disaster (1986) and Deep Water Horizon Oil Spill disaster (2010) highlighted the role of management negligence and the relevance of organizational level, lack of proper knowledge and insincerity shown as pivotal to these catastrophic outcomes [3, 4, 5]. Therefore developing better proactive approach and models to spotlight dangerous occurrences through a veritable decision supportive model becomes necessary. As posited by [6], if inadequate intervention patterns are adopted accidents proliferate.

The Markov chain models have been successfully applied in other fields of research work in engineering. As asserted by [7] Markov method can be powerful tool in reliability, maintainability and safety (RMS) engineering. Markov chains are commonly applies to the study of dependability of complex system. While [8,9], avers to the fact that Markov models are powerful statistical tool and do have a joint history and form a fruitful partnership with maintenance modeling, also been successfully applied in component diagnostics, prognostics and maintenance optimization across a gamut of industries.

Furthermore, the applications of this model in the area of human life safety in the oil and gas sector are limited and therefore need further attention. Markov chain model was applied by [10] in Robot safety identifying potential risk for industrial robot and the definition of hazard rate at different state for robot system. Again [11] reports a fruitful application of Markov chain in predicting risk severity and exposure level of workers in Warri Refining and Petrochemical Company (WRPC) involving four states, with two absorbing states and two non-absorbing states. A survey approach involving that of questionnaire administration instrument to fifty workers was adopted. Again, [12] did a longitudinal study appraising the patterning of episodic incidence of industrial accidents in oil and gas firm in the Niger-delta area of Nigeria using 10-years historical data. Essentially, this study widens the horizon of the application of the Markov chain model covering a 20-State system, with ten absorbing and non-absorbing states respectively.

In general, the study is aimed at unwrapping the deeper meanings, important prediction or implications buried in the industrial accident records by applying Markov-chain model as a

veritable tool that will aid manager's decision in policy making and improving health, safety and environmental performances.

2. Methodology

In an attempt to decipher the deeper meanings buried in the industrial accident data of Nigerian Gas Company Limited (NGC), a subsidiary of Nigerian National Petroleum Company (NNPC), and a Markov chain molded was fitted into a 16-year (2000 – 2015) historic safety data as depicted in Table 1. The industrial accident data were examined for embedded Markov properties namely stochastic regularity, absorbing behaviour and the long-run distribution amongst the various states. The basic assumptions, applicable theorems and formulae leading to the computations and decision making process are precisely stated. Furthermore, the statistical computations were done with the aid of (MATLAB .R. 2016 a) software from which deduction and inference were derived from the results obtained and which guided subsequent discussion and conclusion.

Table 1. *NGC HSE Historical Record from NNPC (2000 – 2015)*

S/N	States	Total	
1.	Fatality (FT)	-	Absorbing States
2.	Third Party Fatality (TFF)	1	
3.	Permanent Disability (PD)	1	
4.	Fire Incidence (severe) (FIs)	23	
5.	Injury (severe) (Is)	13	
6.	Medical Treatment Case (severe) MTCs	165	
7.	Road Traffic Accident (severe) RTAs	-	
8.	Oil Spill (callouts) OS	-	
9.	Restricted Work Case (severe) RWCs	6	
10.	Human Error (HE)	16	
11.	Lost Time Days LTD	30	Non-Absorbing States
12.	Injury (minor)I_m	105	-
13.	Fire Incidence (minor) FI_m	101	
14.	Medical Treatment Case (minor) MTC_m	1202	
15.	Restricted work case (minor) RWC_m	11	
16.	Road Traffic Accident (minor) RTA_m	9	
17.	First Aid Case (FAC)	32	
18.	Unsafe Condition (UC)	365	
19.	Unsafe Acts (UA)	368	
20.	Near Misses (NM)	104	
	Total	2552	

2.1. Theoretical Formulation

(a) Lemmatization

In mathematical theory of probability, a Markov chain is an absorbing chain provided:

Lemma 1: there are one or more absorbing states. Furthermore, a state in a Markov chain is said to be absorbing if the probability of an object leaving the state once entered is zero, or once entered cannot be exited and the probability that it stays in that state is one.
Lemma 2: from each of the non-absorbing states, it is possible to reach some absorbing state in the long run by a number of steps. In an absorbing Markov chain, a state that is not absorbing i.e non-absorbing is also called transient.

(b)Theorem

In standard form, the transition matrix T is expressed as

$$T = \begin{array}{cc} & \begin{array}{cc}\text{Abs} & \text{Non-abs}\end{array} \\ \begin{array}{c}\text{Abs} \\ \text{Non-abs}\end{array} & \left(\begin{array}{c|c} I & O \\ R & Q \end{array}\right) \end{array} \tag{1}$$

Then in standard form:
a. The matrix (I-Q) is invertible.
Where I= identity matrix and Q is non-absorbing matrix. In other words Q is a
Matrix having an initial distribution (or describe the probability of transitioning from some transient state (non-absorbing state) to another.
0 = zero matrix or null matrix, while R matrix describes the probability of transitioning from some non-absorbing state (transients state) to some absorbing state.
For the purpose of this study, we seek to state the long-run distribution of T i.e T^n = T from which the fundamental matrix (N) is derived without proof.

$$T^n = \left(\begin{array}{c|c} I & 0 \\ \hline \left(I + Q + Q^2 + Q^3 + ... + Q^{n-1}\right)R & Q^2 \end{array}\right) \tag{2}$$

The long run distribution of Ṫ

$$T^n = \bar{T} = \left(\begin{array}{c|c} I & O \\ \hline (I - Q)^{-1} R & O \end{array}\right) \tag{3}$$

Equation (iii) is key to the computational analysis that follows, and from which the fundamental matrix N = $(I – Q)^{-1}$ is derived

(c) Fundamental Matrix (N),
N = $(\mathbf{I – Q})^{-1}$ (4)
This is a basic property of an absorbing Markov chain and it represents the expected or average number of visits or number of times objects starts in the i^{th} non-absorbing state to a state(transient) j' before being absorbed.
Primarily, with fundamental matrix (N) in hand, other properties or derivatives are obtained, such as variance on number of visits N_2= N ($2N_{dg}$ – I) - N_{sq}, expected number of step or cumulative movement of object τ = N ξ, variance on number of steps

$\tau_{2=}$ (2N – I) τ - $\tau_{sq,}$ the long run distribution of object or staff among the various absorbing states B = NR and the transient probabilities H= (N – I) N_{dg}^{-1}

(d) Variance on number of visits (N_2),

$$N_2 = N(2N_{dg} - I) - N_{sq} \quad (5)$$

This gives the associated variance on the expected number of visits or simply the variance on the number of visits to a non-absorbing state (transient) j starting at a transient state *i* (before being absorbed) is the (ij)- entry of the matrix.
where in Equation (5), N is the fundamental matrix (N), N_{dg} is diagonal of N with zeroes and N_{sq} is the square of each element of the N matrix.

(e) Expected number of steps or movements (τ)

$$\tau = N\,\xi \quad (6)$$

where $\xi = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ is a column vector whose entries are all unity

The expected number of steps or total number of movements of staff or objects within the non-absorbing state before being absorbed.

(f) Variance on number of step or movements.

$$\tau_2 = (2N - I)\,\tau - \tau_{sq} \quad (7)$$

The variance on the number of steps or movement before being absorbed when starting in a non-absorbing state i. or termed the associated variance, where t_{sq} is the hadamard product of t with itself (i.e. each entry of t is squared).

(g) Absorbing Probabilities (*B*)

$$B = (I - Q)^{-1}R = NR \quad (8)$$

This specifies the long-run distribution of objects among the various absorbing state, provided all object/staff start in the non-absorbing state.

(h) Transient Probabilities (H)

$$H = (N - I)\,N_{dg}^{-1} \quad (9)$$

This gives the probability of likelihood or chances of visiting non-absorbing state j starting at a non-absorbing state i.

2.2. Statistical Computations

The statistical computation when Markov chain is applied involves two (2) regimes viz:

I. the absorbing state regime; and
II. the non-absorbing state regime.

2.2.1. Absorbing State Computations

The diagraph of the 20-states structure in Fig 1 gives an insight on the way the system operates. In this regime all the transition probabilities associated with the absorbing states are heuristically determined. Basically, heuristic algorithm method is adopted which involves reasoning in the determination of the absorbing state probabilities. For example $P_{11} = P_r(FT) = 1$

Mathematically this can be expressed as $P_{11} = P_r(FT) = 1$, implying that a staff who is wasted by fatality (death) remains dead. This is a case of persistence. As stated above, it follows similar heuristic determination applies for others and can be expressed mathematically as $P_{22} = P_{33} = P_{44} \ldots.. = P_{10,10} = 1$. Furthermore, P_{12} which represents a subject transiting from fatality (FT) to third party fatality (TPF); it is a case which is implausible; in other words, this simply means transiting from a state considered absorbing to another state which is an absorbing or non-absorbing is implausible. Then, similar heuristic argument are adopted, $P_{12} = P_{21} = P_{13} = P_{31} = P_{14} = P_{41} = \ldots = P_{1,10} = P_{10,1}$ for reasons of Implausibility.

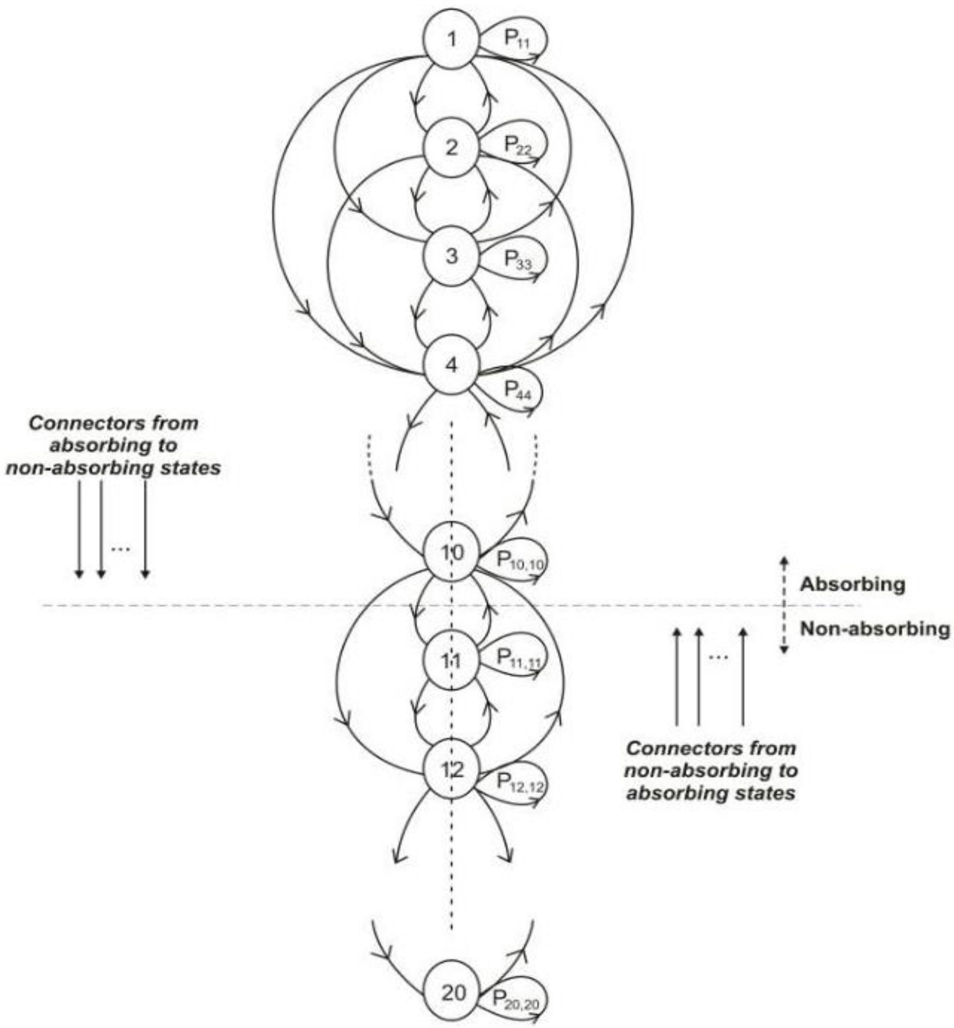

Fig.1. Diagraph of the 20–State structure each state is linked to all the 20-states by an outflow and return lines

2.2.2 Non-absorbing State Computation

In the determination of probabilities under the non-absorbing state regime, Bayesian method is adopted. For this study, ten (10) non-absorbing states are considered namely, Lost Time Days (LTD), Injury (minor) I_m, Fire incidence (minor) FI_m, Medical treatment case (minor) MTC_m, Restricted work case (minor) RWC_m, Road traffic accident (minor) RTA_m, FAC, UC, UA, and NM as shown in Table 1. In the use of Bayesian methodology to compute transition probabilities under non-absorbing regime certain assumptions were considered to allow for mathematic tractability. The assumptions follow.

Assumption 1: A staff that is considered by management to have a lost time day cannot at the same time be available to commit near miss (NM= 0), $P_{11, 20}$.

Assumption 2: A staff who had just suffered a minor injury (I_m) as an entry, cannot be said to have minor fire incidence at the same time (FI_m =0), $P_{12,13}$=0

Assumption 3: A well trained staff that had a minor fire incidence (FI_m) in the industry as recorded cannot be involved in a minor road traffic accident at the same time (a case of impossibility) RTA_m=0, $P_{13,16}$=0.

Assumption 4: A staff whose state is considered as a minor medical treatment case MTC_M cannot be said to be in a state of permanent disability PD =0, $P_{14, 3}$=0.

Assumption 5: A staff who at a particular time is in a state considered as a minor restricted work case (RWC_m) will be unavailable to be exposed to unsafe condition (UC = 0), $P_{15,18}$=0.

Assumption 6: A healthy contract staff who had a minor road traffic accident (RTA_M) cannot transit instantly to third party fatality (death) TPF=0, $P_{16,2}$= 0.

Assumption 7: A staff whose injury is considered to be a first aid case (FAC) cannot be said to have committed a major work related human error (HE = 0), $P_{17,10}$

Assumption 8: It is impossible for a staff to be at a state considered an unsafe condition (UC) at the same time be said to be in severe restricted work case (RWCs = 0), $P_{18,9}$= 0

Assumption 9: A well trained staff who is involved in an unsafe act (UA) cannot instantly transit to a state of severe medical treatment case MTCs = 0, $P_{19,6}$ = 0

Assumption 10: A well trained staff who commits near miss cannot instantly transit to fatality FT= 0, $P_{20,1}$ = 0

Basically, with the ten (10) non-absorbing state considered previously, a ten different combinations of the 20 state structure were arranged in Table 2 with the columns indicating different combination. Furthermore, in applying the Bayesian approach it uses conditional probability of events whose occurrences are presumptuously equated to zero in every column except those whose state total is zero as shown in Table1 represented as serial number 1, 7, 8 which are fatality, road traffic accident severe and oil spill respectively.

Column 1(V_{SET1}) for instance, the probability of subject who commits Unsafe Act (UA) transiting to First Aid Case (FAC), given that the condition that subject has not had near miss earlier, denoted mathematically as $\mathbf{P}\{(UA \rightarrow (FAC)|NM = 0\}$, similarly, in the same column the probability of subject who is exposed to unsafe condition transitioning to the commission of human error, on the account that the subject has not had near miss earlier, is stated thus $P\{(FAC \rightarrow HE|NM= 0)\}$.Similar representations apply for all cases under column 1(V-set 1). Computations for all the sets are collated and depicted in Table 3 also, see Table 4 for mode of transitions

Table 2. *Ten Different Combination of the 20 state structure (NGC) are arrange as follows*

S/NO	V_SET 1	V_SET 2	V_SET 3	V_SET 4	V_SET 5	V_SET 6	V_SET 7	V_SET 8	V_SET 9	V_SET 10
1	FT=0	FT=0	FT=0	FT=0	FT=0	FT=0	FT=0	FT=0	FT=0	FT=0
2	TPF=1	TPF=1	TPF=1	TPF=1	TPF=1	TPF=0	TPF=1	TPF=1	TPF=1	TPF=1
3	PD=1	PD=1	PD=1	PD=0	PD=1	PD=1	PD=1	PD=1	PD=1	PD=1
4	FIs=23	FIs=23	FIs=23	FIs=23	FIs=23	FIs=23	FIs=23	FIs=23	FIs=23	FIs=23
5	Is=13	Is=13	Is=13	Is=13	Is=13	Is=13	Is=13	Is=13	Is=13	Is=13
6	MTCs=165	MTCs=165	MTCs=165	MTCs=165	MTCs=165	MTCs=165	MTCs=165	MTCs=165	MTCs=0	MTCs=165
7	RTAs=0	RTAs=0	RTAs=0	RTAs=0	RTAs=0	RTAs=0	RTAs=0	RTAs=0	RTAs=0	RTAs=0
8	OS_CO=0	OS_CO=0	OS_CO=0	OS_CO=0	OS_CO=0	OS_CO=0	OS_CO=0	OS_CO=0	OS_CO=0	OS_CO=0
9	RWCs=6	RWCs=6	RWCs=6	RWCs=6	RWCs=6	RWCs=6	RWCs=6	RWCs=0	RWCs=6	RWCs=6
10	HE=16	HE=16	HE=16	HE=16	HE=16	HE=16	HE=0	HE=16	HE=16	HE=16
11	LTD=30	LTD=30	LTD=30	LTD=30	LTD=30	LTD=30	LTD=30	LTD=30	LTD=30	LTD=30
12	Im=105	Im=105	Im=105	Im=105	Im=105	Im=105	Im=105	Im=105	Im=105	Im=105
13	FIm=101	FIm=0	FIm=101	FIm=101	FIm=101	FIm=101	FIm=101	FIm=101	FIm=101	FIm=101
14	MTCm=1202	MTCm=1202	MTCm=1202	MTCm=1202	MTCm=1202	MTCm=1202	MTCm=1202	MTCm=1202	MTCm=1202	MTCm=1202
15	RWCm=11	RWCm=11	RWCm=11	RWCm=11	RWCm=11	RWCm=11	RWCm=11	RWCm=11	RWCm=11	RWCm=11
16	RTAm=9	RTAm=9	RTAm=0	RTAm=9	RTAm=9	RTAm=9	RTAm=9	RTAm=9	RTAm=9	RTAm=9
17	FAC=32	FAC=32	FAC=32	FAC=32	FAC=32	FAC=32	FAC=32	FAC=32	FAC=32	FAC=32
18	UC=365	UC=365	UC=365	UC=365	UC=0	UC=365	UC=365	UC=365	UC=365	UC=365
19	UA=368	UA=368	UA=368	UA=368	UA=368	UA=368	UA=368	UA=368	UA=368	UA=368
20	NM=0	NM=104	NM=104	NM=104	NM=104	NM=104	NM=104	NM=104	NM=104	NM=104
TOTAL	2448	2451	2543	2551	2187	2551	2536	2546	2387	2552

In column 5(Vset5): The probability of a subject who had a minor Fire Incidence (FI_m) in row 13 transiting to commission human error HE in column 10 (i-j) entry on the condition that he has not been exposed to an unsafe condition, expressed mathematically, $P\{(FI_M \rightarrow HE)|UC = 0\}$. Similar representation determination for each of the columns is done following similar pattern using conditional probability.

The Bayesian approach essentially uses ratio of subject in any state to the total population of the ten different combinations in Table 2. For instance, column Vset1 serial number 1 fatality (FT) = 0 which a subject state 1, to the ratio of total population 2448. This is expressed mathematically as depicted in Table3 probability set 1. Again, similar computations are done for all Vset1 and other Vsets in Table 2 and mathematically computed to obtain probability set1 and other probability sets in Table3.

Table 3. *Sample Computation of Transition Probabilities using the Bayesian Methodology.*

Probabilities Set1	Probabilities Set2	Probabilities Set3	Probabilities Set4
P(FT)=(0/2448)=0.0000	P(FT)=(0/2451)=0.0000	P(FT)=(0/2543)=0.0000	P(FT)=(0/2551)=0.0000
P(TPF)=(1/2448)=0.0004	P(TPF)=(1/2451)=0.0004	P(TPF)=(1/2543)=0.0004	P(TPF)=(1/2551)=0.0004
P(PD)=(1/2448)=0.0004	P(PD)=(1/2451)=0.0004	P(PD)=(1/2543)=0.0004	P(PD)=(0/2551)=0.0000
P(FI_S)=(23/2448)=0.0094	P(FI_S)=(23/2451)=0.0094	P(FI_S)=(23/2543)=0.0090	P(FI_S)=(23/2551)=0.0090
P(I_S)=(13/2448)=0.0053	P(I_S)=(13/2451)=0.0053	P(I_S)=(13/2543)=0.0051	P(I_S)=(13/2551)=0.0051
P(MTC_S)=(165/2448)=0.0674	P(MTC_S)=(165/2451)=0.0673	P(MTC_S)=(165/2543)=0.0649	P(MTC_S)=(165/2551)=0.0647
P(RTA_S)=(0/2448)=0.0000	P(RTA_S)=(0/2451)=0.0000	P(RTA_S)=(0/2543)=0.0000	P(RTA_S)=(0/2551)=0.0000
P(OS_CO)=(0/2448)=0.0000	P(OS_CO)=(0/2451)=0.0000	P(OS_CO)=(0/2543)=0.0000	P(OS_CO)=(0/2551)=0.0000
P(RWC_S)=(6/2448)=0.0025	P(RWC_S)=(6/2451)=0.0024	P(RWC_S)=(6/2543)=0.0024	P(RWC_S)=(6/2551)=0.0024
P(HE)=(16/2448)=0.0065	P(HE)=(16/2451)=0.0065	P(HE)=(16/2543)=0.0063	P(HE)=(16/2551)=0.0063
P(LTD)=(30/2448)=0.0123	P(LTD)=(30/2451)=0.0122	P(LTD)=(30/2543)=0.0118	P(LTD)=(30/2551)=0.0118
P(I_M)=(105/2448)=0.0429	P(I_M)=(105/2451)=0.0428	P(I_M)=(105/2543)=0.0413	P(I_M)=(105/2551)=0.0412
P(FI_M)=(101/2448)=0.0413	P(FI_M)=(0/2451)=0.0000	P(FI_M)=(101/2543)=0.0397	P(FI_M)=(101/2551)=0.0396
P(MTC_M)=(1202/2448)=0.4910	P(MTC_M)=(1202/2451)=0.4904	P(MTC_M)=(1202/2543)=0.4727	P(MTC_M)=(1202/2551)=0.4712
P(RWC_M)=(11/2448)=0.0045	P(RWC_M)=(11/2451)=0.0045	P(RWC_M)=(11/2543)=0.0043	P(RWC_M)=(11/2551)=0.0043
P(RTA_M)=(9/2448)=0.0037	P(RTA_M)=(9/2451)=0.0037	P(RTA_M)=(0/2543)=0.0000	P(RTA_M)=(9/2551)=0.0035
P(FAC)=(32/2448)=0.0131	P(FAC)=(32/2451)=0.0131	P(FAC)=(32/2543)=0.0126	P(FAC)=(32/2551)=0.0125
P(UC)=(365/2448)=0.1491	P(UC)=(365/2451)=0.1489	P(UC)=(365/2543)=0.1435	P(UC)=(365/2551)=0.1431
P(UA)=(368/2448)=0.1503	P(UA)=(368/2451)=0.1501	P(UA)=(368/2543)=0.1447	P(UA)=(368/2551)=0.1443
P(NM)=(0/2448)=0.0000	P(NM)=(104/2451)=0.0424	P(NM)=(104/2543)=0.0409	P(NM)=(104/2551)=0.0408

Table 4. *NGC mode of Transition Matrix Probability Tableau*

	FT	TPF	PD	Fis	Is	MTCs	RTAs	Osco	RWCs	HE	LTD	Im	Fim	MTCm	RWCm	RTAm	FAC	UC	UA	NM
FT	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TPF	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PD	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Fis	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Is	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MTCs	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
RTAs	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Osco	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
RWCs	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
HE	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
LTD	0	0.0004	0.0004	0.0094	0.0053	0.0674	0	0	0.0025	0.0065	0.0123	0.0429	0.0413	0.491	0.0045	0.0037	0.0131	0.1491	0.1503	0
Im	0	0.0004	0.0004	0.0094	0.0053	0.0673	0	0	0.0024	0.0065	0.0122	0.0428	0	0.4904	0.0045	0.0037	0.0131	0.1489	0.1501	0.0424
Fim	0	0.0004	0.0004	0.009	0.0051	0.0649	0	0	0.0024	0.0063	0.0118	0.0413	0.0397	0.4727	0.0043	0	0.0126	0.1435	0.1447	0.0409
MTCm	0	0.0004	0	0.009	0.0051	0.0647	0	0	0.0024	0.0063	0.0118	0.0412	0.0396	0.4712	0.0043	0.0035	0.0125	0.1431	0.1443	0.0408
RWCm	0	0.0005	0.0005	0.0105	0.0059	0.0754	0	0	0.0027	0.0073	0.0137	0.048	0.0462	0.5496	0.005	0.0041	0.0146	0	0.1683	0.0476
RTAm	0	0	0.0004	0.009	0.0051	0.0647	0	0	0.0024	0.0063	0.0118	0.0412	0.0396	0.4712	0.0043	0.0035	0.0125	0.1431	0.1443	0.0408
FAC	0	0.0004	0.0004	0.0091	0.0051	0.0651	0	0	0.0024	0	0.0118	0.0414	0.0398	0.474	0.0043	0.0035	0.0126	0.1439	0.1451	0.041
UC	0	0.0004	0.0004	0.009	0.0051	0.0648	0	0	0	0.0063	0.0118	0.0412	0.0397	0.4721	0.0043	0.0035	0.0126	0.1434	0.1445	0.0408
UA	0	0.0004	0.0004	0.0096	0.0054	0	0	0	0.0025	0.0067	0.0126	0.044	0.0423	0.5036	0.0046	0.0038	0.0134	0.1529	0.1542	0.0436
NM	0	0.0004	0.0004	0.009	0.0051	0.0647	0	0	0.0024	0.0063	0.0118	0.0411	0.0396	0.471	0.0043	0.0035	0.0125	0.143	0.1442	0.0408

3. Results and Discussion

Standard form of transition matrix

$$T = \left(\begin{array}{c|c} I & O \\ \hline R & Q \end{array}\right)$$

The computed transition probabilities which are consistent with Equation (1) and as displayed in the Table 4, are depicted in the accompanying matrix below..

The Transition probabilities

$$\begin{array}{ccc}
I & & O \\
 & T = \left(\begin{array}{cccccccccc|cccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0.0004 & 0.0004 & 0.0094 & 0.0053 & 0.0674 & 0 & 0 & 0.025 & 0.0065 & 0.0123 & 0.0429 & 0.0413 & 0.491 & 0.0045 & 0.0037 & 0.0131 & 0.1491 & 0.1503 & 0 \\
0 & 0.0004 & 0.0004 & 0.0094 & 0.0053 & 0.0673 & 0 & 0 & 0.024 & 0.0065 & 0.0122 & 0.0428 & 0 & 0.4904 & 0.0045 & 0.0037 & 0.0131 & 0.1489 & 0.1501 & 0.0424 \\
0 & 0.0004 & 0.0004 & 0.009 & 0.0051 & 0.0649 & 0 & 0 & 0.024 & 0.0063 & 0.0118 & 0.0413 & 0.0397 & 0.49727 & 0.0043 & 0 & 0.0126 & 0.1435 & 0.1447 & 0.0409 \\
0 & 0.0004 & 0 & 0.009 & 0.0051 & 0.0647 & 0 & 0 & 0.024 & 0.0063 & 0.0118 & 0.0412 & 0.0396 & 0.4712 & 0.0043 & 0.0035 & 0.0125 & 0.1431 & 0.1443 & 0.0408 \\
0 & 0.0005 & 0.0005 & 0.00105 & 0.0059 & 0.0754 & 0 & 0 & 0.027 & 0.0073 & 0.0137 & 0.048 & 0.0462 & 0.5496 & 0.005 & 0.0041 & 0.0146 & 0 & 0.1683 & 0.0476 \\
0 & 0 & 0.0004 & 0.009 & 0.0051 & 0.0647 & 0 & 0 & 0.024 & 0.0063 & 0.0118 & 0.0412 & 0.0396 & 0.4712 & 0.0043 & 0.0035 & 0.0125 & 0.1431 & 0.1443 & 0.0408 \\
0 & 0.0004 & 0.0004 & 0.0091 & 0.0051 & 0.0651 & 0 & 0 & 0.024 & 0 & 0.0118 & 0.0414 & 0.0398 & 0.474 & 0.0043 & 0.0035 & 0.0126 & 0.1439 & 0.1451 & 0.041 \\
0 & 0.0004 & 0.0004 & 0.009 & 0.0051 & 0.0648 & 0 & 0 & 0 & 0.0063 & 0.0118 & 0.0412 & 0.0397 & 0.4721 & 0.0043 & 0.0035 & 0.0126 & 0.1434 & 0.1445 & 0.0408 \\
0 & 0.0004 & 0.0004 & 0.0096 & 0.0054 & 0 & 0 & 0 & 0.025 & 0.0067 & 0.0126 & 0.044 & 0.0423 & 0.5036 & 0.0046 & 0.0038 & 0.0134 & 0.1529 & 0.1542 & 0.0436 \\
0 & 0.0004 & 0.0004 & 0.009 & 0.0051 & 0.0647 & 0 & 0 & 0.024 & 0.0063 & 0.0118 & 0.0411 & 0.0396 & 0.471 & 0.0043 & 0.0035 & 0.0143 & 0.143 & 0.1442 & 0.0408
\end{array}\right) & \\
R & & Q
\end{array} \qquad (10)$$

3.1 Computation of the fundamental matrix

$$N = (I - Q)^{-1}$$

Computation of the fundamental matrix N as depicted in Equation (4), N = $(I – Q)^{-1}$

$$I - Q = \begin{bmatrix}
-0.9877 & -0.0429 & -0.0413 & -0.491 & -0.0045 & -0.0037 & -0.0131 & -0.1491 & 0.1503 & 0 \\
-0.0122 & 0.9572 & 0 & -0.4904 & -0.0045 & -0.0037 & -0.0131 & -0.1489 & -0.1501 & -0.0424 \\
-0.0118 & -0.0413 & 0.9603 & -0.4727 & -0.0043 & 0 & -0.0126 & -0.1435 & -0.1447 & -0.0409 \\
-0.0118 & -0.0412 & -0.0396 & 0.5288 & -0.5288 & -0.0043 & -0.0125 & -0.1431 & -0.1443 & -0.0408 \\
-0.0137 & -0.048 & -0.0462 & -0.5496 & -0.5496 & 0.995 & -0.0146 & 0 & -0.1683 & -0.0476 \\
-0.0118 & -0.0412 & -0.0396 & -0.4712 & -0.4712 & -0.0043 & -0.0125 & -0.1431 & -0.1443 & -0.0408 \\
-0.0118 & -0.0414 & -0.0398 & -0.474 & -0.474 & -0.0043 & -0.9874 & -0.1439 & -0.1451 & -0.041 \\
-0.0118 & -0.0412 & -0.0397 & -0.4721 & -0.4721 & -0.0043 & -0.0126 & 0.8566 & -0.1445 & -0.0408 \\
-0.0126 & -0.044 & -0.0423 & -0.5036 & -0.5036 & -0.0046 & -0.0134 & -0.1529 & 0.8458 & -0.0436 \\
-0.0118 & -0.0411 & -0.0396 & -0.471 & -0.471 & -0.0043 & -0.0125 & -0.143 & -0.1442 & 0.9592
\end{bmatrix} \qquad (11)$$

$$N = \begin{array}{c} \\ 11 \\ 12 \\ 13 \\ 14 \\ 15(i) \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \end{array} \\ (j) \\ \begin{pmatrix} 1.1515 & 0.5302 & 0.4882 & 6.0698 & 0.555 & 0.0437 & 0.1616 & 1.8339 & 1.8583 & 0.4763 \\ 0.1515 & 1.5302 & 0.447 & 6.0698 & 0.555 & 0.0439 & 0.1616 & 1.8339 & 1.8583 & 0.5187 \\ 0.1515 & 0.5302 & 1.4882 & 6.0698 & 0.555 & 0.0402 & 0.1616 & 1.8339 & 1.8583 & 0.5187 \\ 0.1516 & 0.5304 & 0.4882 & 7.0722 & 0.556 & 0.0437 & 0.1617 & 1.8346 & 1.859 & 0.5189 \\ 0.1514 & 0.53 & 0.4884 & 6.0674 & 0.555 & 0.0437 & 0.1615 & 1.6663 & 1.8576 & 0.5185 \\ 0.1516 & 0.534 & 0.488 & 6.0722 & 0.556 & 1.0437 & 0.1617 & 1.8346 & 1.859 & 0.5189 \\ 0.1524 & 0.5336 & 0.4884 & 6.1081 & 0.559 & 0.044 & 0.1626 & 1.8455 & 1.87 & 0.522 \\ 0.1518 & 0.5315 & 0.4913 & 6.0841 & 0.557 & 0.0438 & 0.162 & 1.8382 & 1.8627 & 0.52 \\ 0.162 & 0.5669 & 0.5219 & 6.4894 & 0.594 & 0.0467 & 0.1728 & 1.9607 & 2.9868 & 0.5546 \\ 0.1515 & 0.5302 & 0.4882 & 6.0698 & 0.555 & 0.0437 & 0.1616 & 1.8339 & 1.8583 & 0.5187 \end{pmatrix} \end{array} \tag{12}$$

The fundamental matrix N will be interpreted alongside the associated variance N_2 or variance of visit as in Equation (5)

3.2 Calculation of Variances and Standard Deviation

Variance on number of visits

$$N_{dg} = \begin{bmatrix} 1.15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.53 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.488 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.07 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.056 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.04 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.163 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.838 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.987 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.519 \end{bmatrix} \tag{13}$$

$$2N_{dg} = \begin{bmatrix} 23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.06 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2976 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 141 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2111 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 209 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2325 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.676 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.974 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.038 \end{bmatrix} \tag{14}$$

$$2N_{dg} - I = \begin{bmatrix} 1.303 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.06 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13.14 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.11 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.088 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.325 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.676 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.974 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.04 \end{bmatrix} \tag{15}$$

$N_{sq} = (N)^2 =$

$$N_{sq} = \begin{pmatrix} 1.3259 & 0.2811 & 0.2383 & 36.8424 & 0.0031 & 0.0019 & 0.0261 & 3.3631 & 3.4533 & 0.2268 \\ 0.0229 & 2.3416 & 0.1998 & 36.8424 & 0.0031 & 0.0019 & 0.0261 & 3.3631 & 3.4533 & 0.2691 \\ 0.0229 & 0.2811 & 2.2147 & 36.8423 & 0.0031 & 0.0016 & 0.0261 & 3.3631 & 3.4533 & 0.2691 \\ 0.023 & 0.2814 & 0.2385 & 50.0156 & 0.0031 & 0.0019 & 0.0261 & 3.3658 & 3.456 & 0.2693 \\ 0.0229 & 0.2809 & 0.2381 & 36.8134 & 1.1141 & 0.0019 & 0.0261 & 2.7765 & 3.4506 & 0.2689 \\ 0.023 & 0.2814 & 0.2385 & 36.8712 & 0.0031 & 1.0894 & 0.0261 & 3.3658 & 3.456 & 0.2693 \\ 0.0232 & 0.2847 & 0.2143 & 37.3087 & 0.0031 & 0.0019 & 1.3517 & 3.4057 & 3.497 & 0.2725 \\ 0.0231 & 0.2825 & 0.2394 & 37.0162 & 0.0031 & 0.0019 & 0.0262 & 8.0554 & 3.4696 & 0.2704 \\ 0.0262 & 0.3213 & 0.2724 & 42.1118 & 0.0035 & 0.0022 & 0.0298 & 3.8442 & 8.9207 & 0.3076 \\ 0.029 & 0.2811 & 0.2383 & 36.8424 & 0.0031 & 0.0019 & 0.0261 & 3.3631 & 3.4533 & 2.3066 \end{pmatrix} \quad (16)$$

The variance of number of visit $N_2 = N\,(2N_{dg} - I) - N_{sq}$ or variance of average number of times:

$$N_2 = (i)\ \begin{array}{c|c} & \begin{array}{cccccccccc} 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \end{array} \\ & (j) \\ \begin{array}{c} 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} & \begin{pmatrix} 0.1744 & 0.8114 & 0.7265 & 42.941 & 0.0586 & 0.0456 & 0.188 & 5.2129 & 5.789 & 0.7435 \\ 0.1744 & 0.8114 & 0.6836 & 42.941 & 0.0586 & 0.0458 & 0.188 & 5.2129 & 5.789 & 0.7878 \\ 0.1744 & 0.8114 & 0.7265 & 42.941 & 0.0586 & 0.0421 & 0.188 & 5.2129 & 5.789 & 0.7878 \\ 0.1745 & 0.8116 & 07267 & 42.943 & 0.0587 & 0.0457 & 0.1881 & 5.2136 & 5.7899 & 0.788 \\ 0.1744 & 0.8112 & 0.7263 & 42.939 & 0.0586 & 0.0456 & 0.188 & 5.0157 & 5.7881 & 0.7876 \\ 0.1745 & 0.8116 & 0.7267 & 42.943 & 0.0587 & 0.0457 & 0.1881 & 5.2136 & 5.7899 & 0.788 \\ 0.1754 & 0.8147 & 0.7296 & 42.978 & 0.059 & 0.0459 & 0.1891 & 5.2244 & 5.8036 & 0.7911 \\ 0.1748 & 0.8126 & 0.7276 & 42.955 & 0.0588 & 0.0457 & 0.1884 & 5.2172 & 5.7945 & 0.789 \\ 0.1848 & 0.8467 & 0.7591 & 43.187 & 0.0625 & 0.0486 & 0.1991 & 5.3147 & 5.934 & 0.8224 \\ 0.1744 & 0.8114 & 0.7265 & 42.941 & 0.0586 & 0.0456 & 0.188 & 5.2129 & 5.789 & 0.7878 \end{pmatrix} \end{array} \quad (17)$$

3.3 Interpretation of the fundamental matrix N Eq.(4), and its derivative N_2.

Essentially, as discussed earlier, N estimates the number of times or visit subjects (staff) starting from any of the non-absorbing state, transit to other states within the transient state before being absorbed, with the associated variability estimate as given by matrix – N_2 will be interpreted together.

Sample interpretation from N and N_2 as in Eq(12) and Eq(17) respectively.

(i) 20 → 12 (Near miss (Nm) → Minor Injury I_m)
$N = 0.5302$, $N_2 = 0.8114$

The significance of this entry is that for every 1,000 near miss visit in transiting to minor injury (I_m), 530 times of the near misses results into minor injury (I_m) if the safety function remains constant. This can be interpreted also that there is 53% possibility of every near miss leading to a minor injury (I_m). The event happens with an associated variance of 0.8114 and a standard deviation $\sigma = \sqrt{0.8114} = 0.9008 \in N_2$

(ii) 19 → 12 (Unsafe Act (UA) → Minor Injury (I_m))
$N = 0.5669$, $N_2 = 0.8469$

This entry signifies that for instance every 1,000 visits or number of time of Unsafe Act transiting to minor Injury, 567 times will end up in minor injury (I_m) or 57%. This is expected because of the volatility of the gases environment. This happens with an associated variance 0.8469, standard deviation $\sigma = \sqrt{0.8469} = 0.9202 \in N_2$

Similarly

(i) 18 → 17 (N,8,17) (Unsafe condition (UC) → First Aid Case(FAC)
$N_{18,17} = 0.162$, $N_2(_{18,17}) = 0.1884$

The significance of this entry is that in every 100 times of unsafe condition exposure, 16 of such exposure will lead to field staff sustaining injury that will require first aid treatment. This happens with an associated variance of N_2 = 0.1884 and a standard deviation σ = $\sqrt{0.1884}$ = 0.434 ∈ N_2

Similarly, other essential inferences can be deduced from the fundamental matrix N and its associated variance matrix N_2 for other state parameter of interest.

Computation and interpretation of the absorbing state matrix

$B = (I - Q)^{-1} R = NR$ as in Equation (8), N and R matrices were stated earlier.

$$
\begin{array}{ccc}
 & & \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\
 & & (j) \\
B = NR = & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} (i) &
\begin{pmatrix}
0 & 0.005 & 0.0027 & 0.1161 & 0.0656 & 0.7048 & 0 & 0 & 0.026 & 0.0798 \\
0 & 0.005 & 0.0027 & 0.1161 & 0.0656 & 0.7048 & 0 & 0 & 0.026 & 0.0798 \\
0 & 0.005 & 0.0027 & 0.1161 & 0.0656 & 0.7048 & 0 & 0 & 0.026 & 0.0798 \\
0 & 0.005 & 0.0023 & 0.1162 & 0.0657 & 0.705 & 0 & 0 & 0.026 & 0.0798 \\
0 & 0.005 & 0.0027 & 0.1161 & 0.0656 & 0.7045 & 0 & 0 & 0.0264 & 0.0797 \\
0 & 0.0046 & 0.0027 & 0.1162 & 0.0657 & 0.705 & 0 & 0 & 0.026 & 0.0798 \\
0 & 0.0051 & 0.0027 & 0.1169 & 0.0661 & 0.7092 & 0 & 0 & 0.0261 & 0.074 \\
0 & 0.005 & 0.0027 & 0.1164 & 0.0658 & 0.7064 & 0 & 0 & 0.0237 & 0.08 \\
0 & 0.0054 & 0.0029 & 0.1242 & 0.0702 & 0.6843 & 0 & 0 & 0.0278 & 0.0853 \\
0 & 0.005 & 0.0027 & 0.1161 & 0.0656 & 0.7048 & 0 & 0 & 0.026 & 0.0798
\end{pmatrix}
\end{array}
\qquad (18)
$$

$$
B = \begin{pmatrix}
Fatality & \to & 0 & Third\ Party\ Fatality & \to & 0.005 \\
Permanent\ Disability & \to & 0.0027 & Fire\ Incidence(Severe) & \to & 0.1161 \\
Injury(Severe) & \to & 0.0656 & Medical\ Treatment\ Case(Severe) & \to & 0.7048 \\
Road\ Trafic\ Accident(Severe0 & \to & 0 & Oil\ Spill\ (Call\ Outs) & \to & 0 \\
\mathrm{Re}\ stricted\ Work\ Cases(Severe) & \to & 0.026 & Human\ Error & \to & 0.0798
\end{pmatrix}
\qquad [19]
$$

The numbering of matrices N, N_2 and B is in consonant with the states as depicted in Table 1 which facilitates cross references. The B matrix gives the long-run transition of subject (staff) within the system which shows the general trend. It is evident that the row entries in each column are the same showing that it is actually represents a stabilized matrix (trend) and thus, can be interpreted column wise. Column 1 represents state 1 which is fatality, while column 2 represents state 2 which is third party fatality, just to mention a few. For example, column wise, column 6 which represents medical treatment case Severe (MTC_S) reveals that 704 subjects in every 1,000 are going to be categorized to have a severe medical condition which requires treatment i.e. (medical treatment case severe) if the trend of injury, illness and gaseous exposure remain the same. While fire incidence severe in column 4 implies that in every 100 subject, about 11.6 i.e. 12 will lead to severe fire incidence (FI_s).Plausible too!

The accompanying graphical representation in matrix Equation (18) maps states to the long-run probabilities of occurrence as discernable from the B-matrix depicted supra.

The total movements of subject within the non-absorbing state before being absorbed is computed as follow. This movement is represented by the τ-matrix given by

$$\tau = \begin{pmatrix} 1.152 & 0.53 & 0.49 & 6.07 & 0.056 & 0.04 & 0.162 & 1.83 & 1.858 & 0.476 \\ 0.152 & 1.53 & 0.45 & 6.07 & 0.056 & 0.04 & 0.162 & 1.83 & 1.858 & 0.519 \\ 0.152 & 0.53 & 1.49 & 6.07 & 0.056 & 0.04 & 0.162 & 1.83 & 1.858 & 0.519 \\ 0.152 & 0.53 & 0.49 & 7.072 & 0.056 & 0.04 & 0.162 & 1.83 & 1.859 & 0.519 \\ 0.151 & 0.53 & 0.49 & 6.067 & 1.056 & 0.04 & 0.162 & 1.67 & 1.858 & 0.519 \\ 0.152 & 0.53 & 0.49 & 6.072 & 0.056 & 1.04 & 0.162 & 1.83 & 1.859 & 0.519 \\ 0.152 & 0.53 & 0.49 & 6.108 & 0.056 & 0.04 & 1.163 & 1.85 & 1.87 & 0.522 \\ 0.152 & 0.53 & 0.49 & 6.084 & 0.056 & 0.04 & 0.162 & 2.84 & 1.863 & 0.52 \\ 0.162 & 0.57 & 0.52 & 6.489 & 0.059 & 0.05 & 0.173 & 1.96 & 2.987 & 0.555 \\ 0.152 & 0.53 & 0.49 & 6.07 & 0.056 & 0.04 & 0.162 & 1.83 & 1.858 & 1.519 \end{pmatrix} x \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 12.669 \\ 12.67 \\ 12.708 \\ 12.716 \\ 12..54 \\ 12.716 \\ 12.785 \\ 12.739 \\ 13.521 \\ 12.712 \end{pmatrix} \quad (20)$$

And its associated variance is τ_2 as in Eq (7) and matrix Equation (22)

τ_2= (2N – I) τ - τ_{sq}

$$(2N-1)\tau - \tau_{sq} = \begin{pmatrix} 1.303 & 1.06 & 0.98 & 12.14 & 0.111 & 0.09 & 0.323 & 3.67 & 3.717 & 0.953 \\ 0.303 & 2.06 & 0.89 & 12.14 & 0.111 & 0.09 & 0.323 & 3.67 & 3.717 & 1.038 \\ 0.303 & 1.06 & 1.98 & 12.14 & 0.111 & 0.08 & 0.323 & 3.67 & 3.717 & 1.038 \\ 0.303 & 1.06 & 0.98 & 13.14 & 0.111 & 0.09 & 0.323 & 3.67 & 3.718 & 1.038 \\ 0.303 & 1.06 & 0.98 & 12.13 & 1.111 & 0.09 & 0.323 & 3.33 & 3.715 & 1.037 \\ 0.303 & 1.06 & 0.98 & 12.14 & 0.111 & 1.09 & 0.323 & 3.67 & 3.718 & 1.038 \\ 0.305 & 1.07 & 0.98 & 12.22 & 0.112 & 0.09 & 1.325 & 3.69 & 3.74 & 1.044 \\ 0.304 & 1.06 & 0.98 & 12.17 & 0.111 & 0.09 & 0.324 & 4.68 & 3.725 & 1.04 \\ 0.324 & 1.13 & 1.04 & 12.98 & 0.119 & 0.09 & 0.346 & 3.92 & 4.974 & 1.109 \\ 0.303 & 1.06 & 0.98 & 12.14 & 0.111 & 0.09 & 0.323 & 3.67 & 3.717 & 2.038 \end{pmatrix} x \begin{pmatrix} 12.669 \\ 12.6704 \\ 12.7079 \\ 12.7161 \\ 12.54 \\ 12.7161 \\ 12.7854 \\ 12.7391 \\ 13.521 \\ 12.7115 \end{pmatrix}$$

$$- \begin{pmatrix} 160.503 \\ 160.539 \\ 161.491 \\ 161.698 \\ 157.251 \\ 161.698 \\ 163.465 \\ 162.284 \\ 182.818 \\ 161.581 \end{pmatrix} = \begin{pmatrix} 151.9367 \\ 151.9386 \\ 151.9774 \\ 151.9868 \\ 151.7697 \\ 151.9868 \\ 152.0691 \\ 125.0152 \\ 152.3503 \\ 151.981 \end{pmatrix} \quad (21)$$

$$\tau_2 = \begin{pmatrix} 151.9367 \\ 151.9386 \\ 151.9774 \\ 151.9868 \\ 151.7697 \\ 151.9868 \\ 152.0691 \\ 125.0152 \\ 152.3503 \\ 151.981 \end{pmatrix} \quad (22)$$

$$\text{Standard deviation } \sigma = \sqrt{\tau_2} = \begin{pmatrix} 12.3263 \\ 12.3263 \\ 12.3279 \\ 12.3283 \\ 12.3195 \\ 12.3283 \\ 12.3316 \\ 12.3294 \\ 12.343 \\ 12.3281 \end{pmatrix} \quad (23)$$

The Eq.(20) for τ suggests that subjects on the average change position or habituate about 13times among the non-absorbing states before being finally absorbed (trapped) into any of the ten absorbing state numbering 1-10 in Table 1. While the accompanying matrix (xxii) estimates the associated variance expected number of movement or habituation τ_2 = 152 and standard deviation Eq (23) of about 12. This implies that the estimation of N and B computed could hover about this mean value of 12.

Transient probability of visiting transient states refer to Equation (9) H = (N - I) Ndg^{-1}

Transient probabilities of visiting transient states $H = (N - I)N_{dg}^{-1}$

$$N - I = \begin{pmatrix} 0.1515 & 0.5302 & 0.4882 & 6.0698 & 0.0555 & 0.0437 & 0.1616 & 1.8339 & 1.8583 & 0.4763 \\ 0.1515 & 0.5302 & 0.447 & 6.0698 & 0.0555 & 0.0439 & 0.1616 & 1.8339 & 1.8583 & 0.5187 \\ 0.1515 & 0.5302 & 0.4882 & 6.0698 & 0.0555 & 0.0402 & 0.11616 & 1.8339 & 1.8583 & 0.5187 \\ 0.1516 & 0.5304 & 0.4884 & 6.0722 & 0.0556 & 0.0437 & 0.1617 & 1.8346 & 1.859 & 0.5189 \\ 0.1514 & 0.53 & 0.488 & 6.0674 & 0.0555 & 0.0437 & 0.1615 & 1.6663 & 1.8576 & 0.5185 \\ 0.1516 & 0.5304 & 0.4884 & 6.0722 & 0.0556 & 0.0437 & 0.1617 & 1.8346 & 1.859 & 0.5189 \\ 0.1524 & 0.5336 & 0.4913 & 6.1081 & 0.0559 & 0.044 & 01626 & 1.8455 & 1.87 & 0.522 \\ 0.1518 & 0.5315 & 0.4893 & 6.0841 & 0.0557 & 0.0438 & 0.126 & 1.8382 & 1.8627 & 0.52 \\ 0.162 & 0.5669 & 0.5219 & 6.4894 & 0.0594 & 0.0467 & 0.1728 & 19607 & 1.9868 & 0.5546 \\ 0.1515 & 0.5302 & 0.4882 & 6.0698 & 0.0555 & 0.0437 & 0.1616 & 1.8339 & 1.8583 & 0.5187 \end{pmatrix} \quad (24)$$

$$H = \begin{array}{c} \\ \\ (i) \end{array} \begin{array}{c|c} & \begin{array}{cccccccccc} 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \end{array} \\ & (j) \\ \begin{array}{c} 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} & \begin{pmatrix} 0.1316 & 0.3456 & 0.328 & 0.8583 & 0.0526 & 0.0419 & 0.139 & 0.6461 & 0.6222 & 0.3136 \\ 0.1316 & 0.3465 & 0.3003 & 0.8583 & 0.0526 & 0.042 & 0.139 & 0.6461 & 0.6222 & 0.3416 \\ 0.1316 & 0.3465 & 0.328 & 0.8583 & 0.0526 & 0.0385 & 0.139 & 0.6461 & 0.6222 & 0.3416 \\ 0.1316 & 0.3466 & 0.3282 & 0.8586 & 0.0526 & 0.0419 & 0.139 & 0.6464 & 0.6224 & 0.3417 \\ 0.1315 & 0.3464 & 0.3279 & 0.8579 & 0.0526 & 0.0419 & 0.1389 & 0.5871 & 0.6219 & 0.3414 \\ 0.1316 & 0.3466 & 0.3282 & 0.8586 & 0.0526 & 0.0419 & 0.139 & 0.6464 & 0.6224 & 0.3417 \\ 0.1324 & 0.3487 & 0.3301 & 0.8637 & 0.053 & 0.0422 & 0.1399 & 0.6502 & 0.6261 & 0.3437 \\ 0.1319 & 0.3473 & 0.3288 & 0.8603 & 0.0527 & 0.042 & 0.1393 & 0.6477 & 0.6236 & 0.3424 \\ 0.1407 & 0.3705 & 0.3507 & 0.9176 & 0.0563 & 0.0448 & 0.1486 & 0.6908 & 0.6652 & 0.3652 \\ 0.1316 & 0.3465 & 0.328 & 0.8583 & 0.0526 & 0.0419 & 0.139 & 0.6461 & 0.6222 & 0.3416 \end{pmatrix} \end{array} \quad (25)$$

The H- matrix as in matrix Equation (25) estimates the probability (chances) of a subject transiting amongst the non-absorbing state. For instance $H_{20,\,14}$ = 0.8583 the transient to transient estimate or likelihood probability of visit from near misses to medical treatment case minor (MTC_m) has 85 chances of occurrence in every 100.

Furthermore, $H_{19,\,19}$ = 0.665, the transient to transient probability visit to unsafe acts give 67% chance or likelihood probability which agrees with the high state value total in Table 1

4. Conclusion

Safety performance function is vital to the growth and survival of all engineering firm in term of increase productivity, cutting edge competitive advantage and over all safety of the employee and the environment. This can be achieved by applying better approaches and model in predicting accident outcomes which the Markov chain model comes handy.

Results from the study suggest certain trend and pattern as depicted in the long-run transition distribution safety matrix B = NR as subjects transit or habituate from a non-absorbing state

before entering any of the absorbing states. Fatality, permanent disability, severe injury was significantly low. Although, there are other absorbing state parameters that are of concern as evident in the study, such as human error (HE) 8% with an attendant severe fire incident occurrence of 11.6% because of volatility (gas) of the working environment, which signals a worrisome development. Remarkably, 70.5% of the field workers had severe medical treatment case (MTCs) over a 16-year period traceable to occupational injury and illness due to poisonous inhalation of gases.

Consequently, the significance of the above import of severe medical treatment case to the organization under study is weighty in terms of cost of medical bill, reduced productivity, lost time days and increase workload for staff replacing the affected co-worker. From the foregoing managers can gain deeper insight of incident level from the application of the Markov chain model as a veritable tool for decision making. For instance, the transient to transient movement (H) and other transitions or habituation to absorbing states made by workers can be used to signal impending danger or can be shown on bill board as caution to workers on how they transit or habituate in the various states before entrapped in an absorbing state, with the right remediation in place.

In summary, the Markov chain model offers a veritable `framework for the development of safety system for similar organization.

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